A MODEL OF THE COUPLING BETWEEN POSTS IN WAVEGUIDES USING EQUIVALENT TRANSMISSION LINES

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ABSTRACT

In this paper we show that the model for the coupling between posts in waveguides orignally proposed by Joshi and Cornick[2,3], has an equivalent version that uses transmission lines for each mode pair. With this model, the problem is solved based on travelling waves and may be useful for the solution of problems with multiple posts without the necessity of having to obtain a Green's function that would include all boundaries. We also show that we may reduce by one half the number of terms of the expansion of the current density induced in the post, as suggested by Eisenhart and Khan[1].

Introduction

Coupling between posts in waveguides has been studied in several papers[2,3,4], being apparent that one of the main problems is to derive and apply a Green's function that take into account the ends of the waveguide and the presence of obstacles between posts. The solution represents equivalent circuits based in lumped elements whose complexity difficults their interpretation in simple terms, even when they are right. It is desirable to be able to interconnect elements in structures of this type without having to resort to a theoretical solution for each case, i.e., to be able to use solutions for more complex cases.

This type of problems can be worked out interpreting the already known solution in therms of transmission lines, rather than lumped elements, that is to say to consider directly traveling waves.

This paper shows that the already known solutions can be interpreted in terms of transmission lines, which propagate the excited waves, by means of arrays of posts, and ideal transformers, these ones being already identified in the previous works as coupling the geometry of the post to such lines.

Derivation of the Green's Function

The required Green's function to evaluate the immitance elements[2,3], is the one which corresponds to the electric field intensity constrained to the $\hat{y}\hat{y}$ component for rectangular cross section waveguides, as shown in Figure 1.

When there is no reflection on planes t_1 and t_2, the Green's function is given by[1]

$$\vec{\vec{G}}_E = \hat{y}\hat{y} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{(2-\delta_n)(k^2-k_y^2)}{abk^2\Gamma_{mn}} \sin k_x x \cos k_y y \sin k_x x^1 \cos k_y y^1 e^{-\Gamma_{mn}|z-z^1|} \quad (1)$$

Expression (1) is a solution developed in terms of ortogonal modes.

If only a mode pair m=i,n-j, is considered, then the case for a waveguide arbitrarily ended can be modeled by means of a transmission line, as it is shown in Figure 2.

The representation by means of this model is possible only because the boundary planes represented by t_1 and t_2 produce only wave reflection with the Γ_{ij} propagation constant.

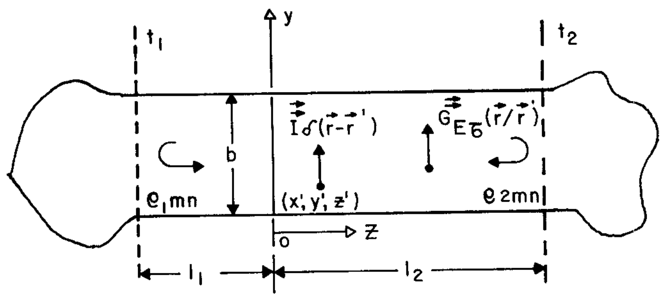

FIGURE 1. WAVEGUIDE SHOWING ARBITRARY LOADS ON PLANES t_2 AND t_2, EXCITED BY AN ELECTRIC CURRENT IMPULSE $\vec{\vec{I}}\delta(\vec{r}-\vec{r}^1)$ and GIVING A RESPONSE $\vec{\vec{G}}_{ET}(\vec{r}/\vec{r}^1)$, ρ_{1mn} AND ρ_{2mn} ARE THE REFLECTION COEFFICIENTS FOR EACH MODE PAIR IN PLANES t_1 AND t_2, RESPECTIVELY.

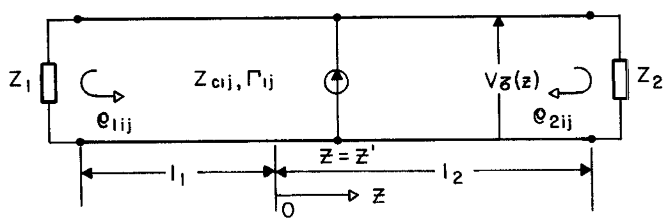

FIGURE 2. MODEL OF THE CASE PRESENTED IN FIGURE 1, FEATURING THE TRANSMISSION LINE, WHEN THE MODE PAIR (i,j) IS EXCITED.

The solution found for the $V_T(z)$ induced tension, using classic method in transmission lines, is:

$$V_T(z) = \frac{IZ_{cij}}{2} \left(\frac{1+S_1+S_2+S_1S_2}{1-S_1S_2 e^{-2\Gamma_{ij}|z-z^1|}} \right) e^{-\Gamma_{ij}|z-z^1|}$$

where

$$S_1 = \rho_{1ij} e^{\Gamma_{ij}(|z-z^1|-(z+z^1)-2\ell_1)} \quad (2.1)$$

$$S_2 = \rho_{2ij} e^{\Gamma_{ij}(|z-z^1|+(z+z^1)-2\ell_2)} \quad (2.2)$$

When $Z_1=Z_2=Z_{cij}$, $\rho_{1ij} = \rho_{2ij} = 0$, $V_T(z)$ is reduced to

$$V(z) = \frac{IZ_{cij}}{2} e^{-\Gamma_{ij}|z-z^1|}$$

These expressions shown that Green's function for the mode pair (i,j) with mismatched loads in both ends of the waveguide is given by

$$\vec{\vec{G}}_{E\tau} = \hat{y}\hat{y}\left\{\frac{(2-\delta_n)(k^2-k_y^2)}{abk^2\Gamma_{mn}} \sin k_x x \cos k_y y \; \sin k_x x^1 \cos k_y y^1 e^{-\Gamma_{mn}|z-z^1|} \left(\frac{1+S_1+S_2+S_1S_2}{1-S_1S_2 e^{-2\Gamma_{mn}|z-z^1|}}\right)\right\}_{\substack{m=i\\n=j}}$$

The solution for the general case, taking into account the mode orthogonality and following our model is

$$\vec{\vec{G}}_{E\tau}(\vec{r}/\vec{r}^1) = \hat{y}\hat{y}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{(2-\delta_n)(k^2-k_y^2)}{abk^2\Gamma_{mn}} \sin k_x x \cos k_y y \; \sin k_x x^1 \cos k_y y^1 e^{-\Gamma_{mn}|z-z^1|} \left(\frac{1+S_1+S_2+S_1S_2}{1-S_1S_2 e^{-2\Gamma_{mn}|z-z^1|}}\right) \quad (3)$$

Expansion of $\vec{J}(\vec{r})$

Current density induced at the posts[1,2] has the general form

$$\vec{J}_i(\vec{r}) = \hat{y}J_{oi}u(y)u(x)\delta(z-z_i)$$

where

$$u(y) = \sum_{\ell=0}^{\infty} \frac{2-\delta_\ell}{2b}\left(A_{i\ell}^{\,y}\cos\frac{\ell\Pi y}{b} + B_{i\ell}^{\,y}\sin\frac{\ell\Pi y}{b}\right) \quad (4.1)$$

and

$$u(x) = \sum_{f=0}^{\infty} \frac{2-\delta_f}{2a}\left(A_{if}^{\,x}\cos\frac{f\Pi x}{a} + B_{if}^{\,x}\sin\frac{f\Pi x}{b}\right) \quad (4.2)$$

Since the description of function u(x) and u(y) at intervals $0\leq x\leq a$ and $0\leq y\leq b$ is the only one that interests us, we can described them by either the odd or the even pair of expressions (4.1) and (4.2).

We have chosen functions

$$u(y) = \sum_{\ell=0}^{\infty} \frac{2-\delta_\ell}{2b} A_{i\ell}^{\,y}\cos\frac{\ell\Pi y}{b} \quad (5.1)$$

$$u(x) = \sum_{f=1}^{\infty} \frac{1}{a} B_{if}^{\,x}\sin\frac{f\Pi x}{a} \quad (5.2)$$

on the base that they are consistent, term by term, with the type of image that electric current reflect on the electric walls (transversal wall in the case of (5.1) and parallel wall for (5.2)).

With this expansion, we obtain the same model for the coupling between posts as the obtained by Joshi and Cornick[3], but with much more simplicity since we used half of the terms.

Coupling Between Posts Using Equivalent Transmission Lines

Eisenhart and Khan showed that the pair modes impedance for one post in mismatched waveguides loads is the same for two adapted loads multiplied by a factor. We have directly obtained it from the last factor in parenthesis in (3) whose value is given by:

$$\tau = \frac{1+\rho_{1mn}e^{-2\Gamma_{mn}\ell_1}+\rho_{2mn}e^{-2\Gamma_{mn}\ell_2}+\rho_{1mn}\rho_{2mn}e^{-2\Gamma_{mn}(\ell_1+\ell_2)}}{1-\rho_{1mn}\rho_{2mn}e^{-2\Gamma_{mn}(\ell_1+\ell_2)}} \quad (6)$$

so that

$$z_{Tmn} = z_{mn}\tau$$

where:

z_{mn} = Mode pair impedance (m,n) for matched load waveguide.
z_{Tmn} = Mode pair impedance (m,n) for mismatched load waveguide.

They also indicated that the factor takes the interesting and expected form of two combined parallel transmission lines.

Joshi and Cornick also have showed[3] that, from their lumped elements model for the coupling between two posts with one gap in each post, the corresponding quadripole coupling for the H_{10} mode is equal in the symetric case ($k_{1p_1} = k_{2p_1}$) to the expressions for the transmission line of length L, except for the $k_{p_1}^{\,2}$ factor.

In fact, it is possible to obtain this length of transmission line, we performed it in the following way.

The short circuit admitances for the (m,n) mode pair according to Joshi and Cornick model for staggered two posts, with matched waveguide are:

$$Y_{11} = 1/\left(Z_{mn}(k_{1pm}/k_{1gn})^2(1-e^{-2\Gamma_{mn}L_{12}})\right)$$

$$Y_{12} = -e^{-\Gamma_{mn}L_{12}}/\left(Z_{mn}(k_{1pm}/k_{1gn})(k_{2pm}/k_{2gn})(1-e^{-2\Gamma_{mn}L_{12}})\right)$$

$$Y_{22} = 1/\left(Z_{mn}\left(\frac{k_{2pm}}{k_{2gn}}\right)^2(1-e^{-2\Gamma_{mn}L_{12}})\right)$$

where

L_{12} = Length between posts.
$Z_{cmn} = 2Z_{mn}$

and these can be represented by the equivalents circuits of Figure 3.

If the circuit loads Z_{cmn} of Figure 3 are considered as transmission lines of characteristic impedance Z_{cmn} and infinite lengths, it can be concluded that the equivalent circuit with mismatched loads at the guide ends, for the coupling in (m,n) mode pair between posts, is that of Figure 4.

This same conclusion follows from the one post model of Eisenhart and Khan, by interpreting the impedance for the mode pair as the parallel of two transmission lines.

For the complete two post case, the only thing we have to do is to connect pieces of transmission lines with length L_{12} propagating constant Γ_{mn}, and characteristic impedance $Z_{cmn} = 2Z_{mn}$, with their ends toward mode pair (m,n) terminals, as in the Eisenhart and Khan model. The other transmission piece at each post can be ended in any load, which results equivalent to the loads and modifications given by Joshi and Cornick[4].

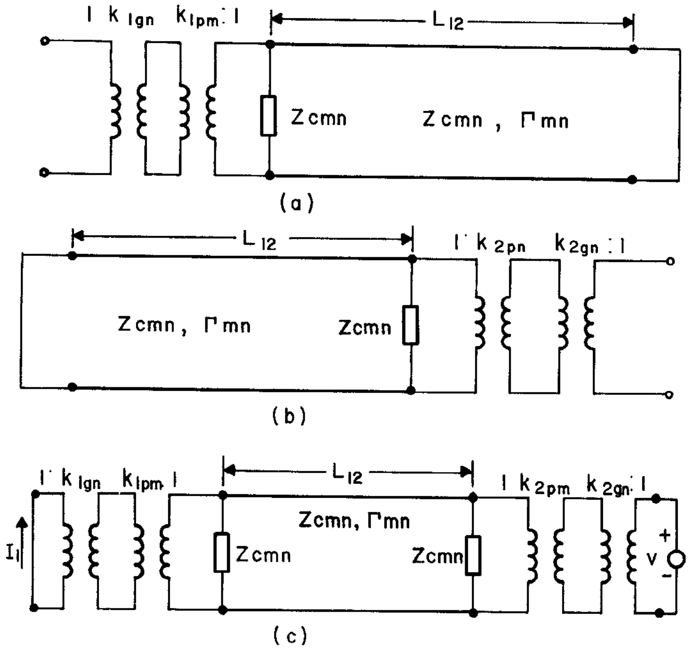

FIGURE 3. EQUIVALENT CIRCUITS FOR THE SHORT CIRCUIT ADMITTANCES (a) Y_{11}; (b) Y_{22}; (c) $Y_{12} = Y_{21}$.

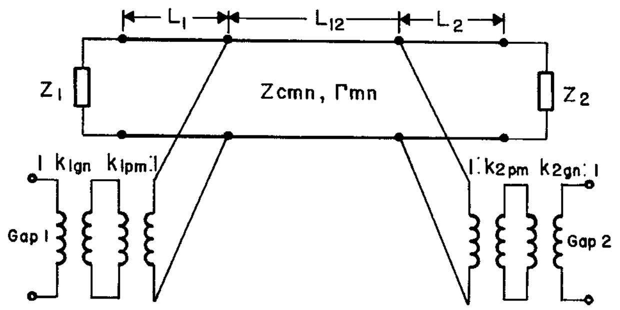

FIGURE 1. EQUIVALENT CIRCUIT FOR THE MODE PAIR (m,n), SHOWING THE WAVEGUIDE AS A TRANSMISSION LINE AND WITH MISMATCHED LOADS AT THE ENDS.

Conclusions

We have interpreted the coupling among posts in waveguides in terms of equivalent transmission lines. The approach was based in the application of Green's functions of the form given at expression (3) and interpreted in terms of the model on Figure 2. This last interpretation was proved by derivating the equivalent transmission line that couples two posts in (m,n) mode pair, taking the lumped model of Joshi and Cornick as the beginning, and by observing that is similar to the result obtained from the one post model with one gap when the mode pair impedance is interpreted as a combination of transmission lines. The complete analysis was expedited by using the expansions for the current density given in expression (5).

This interpretation can be used, for example, for the single cavity-multiple-device analysis.

References

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